CHAPTER OUTLINE

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INTRODUCTION

The task of managing major projects is an ancient and honorable art. In about 2600 B.C., the Egyptians built the Great Pyramid for King Khufu. The Greek historian Herodotus claimed that 400,000 men worked for 20 years to build this structure. Although these figures are now in doubt, there is no question about the enormity of the project. The Book of Genesis reports that the Tower of Babel was not completed because God made it impossible for the builders to communicate. This project is especially important, since it establishes a historical precedent for the ever-popular practice of citing divine intervention as a rationale for failure.¹

Modern projects ranging from building a suburban shopping center to putting a man on the moon are amazingly large, complex, and costly. Completing such projects on time

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¹ The Chicago Tribune (August 5, 1977) noted the following comment concerning the blackout in New York in July of that year: “Con Ed called the disaster an act of God.”
and within the budget is not an easy task. In particular, we shall see that the complicated
problems of scheduling such projects are often structured by the interdependence of activ-
ities. Typically, certain of the activities may not be initiated before others have been com-
pleted. In dealing with projects possibly involving thousands of such dependency relations,
it is no wonder that managers seek effective methods of analysis. Some of the key questions
to be answered in this chapter are

1. What is the expected project completion date?
2. What is the potential “variability” in this date?
3. What are the scheduled start and completion dates for each specific activity?
4. What activities are critical in the sense that they must be completed exactly as sched-
   uled in order to meet the target for overall project completion?
5. How long can noncritical activities be delayed before a delay in the overall completion
date is incurred?
6. How might resources be concentrated most effectively on activities in order to speed
   up project completion?
7. What controls can be exercised on the flows of expenditures for the various activities
   throughout the duration of the project in order that the overall budget can be
   adhered to?

**PERT** and **CPM**, acronyms for Program Evaluation Review Technique and Critical
Path Method, respectively, will provide answers to these questions. Each of these
approaches to scheduling represents a project as a network, and hence the material in this
chapter can be viewed as an extension of the deterministic networks discussed in Chapter
5. When a project involves uncertain elements, the representation of the project requires a
stochastic network, which introduces an additional level of complexity not present in
Chapter 5.

PERT was developed in the late 1950s by the Navy Special Projects Office in coopera-
tion with the management consulting firm of Booz, Allen, and Hamilton. The technique
received substantial favorable publicity for its use in the engineering and development pro-
gram of the Polaris missile, a complicated project that had 250 prime contractors and over
9,000 subcontractors. Since that time, it has been widely adopted in other branches of gov-
ernment and in industry and has been applied to such diverse projects as construction of
factories, buildings, and highways, research management, product development, the instal-
lation of new computer systems, and so on. Today, many firms and government agencies
require all contractors to use PERT.

CPM was developed in 1957 by J. E. Kelly of Remington Rand and M. R. Walker of
Du Pont. It differs from PERT primarily in the details of how time and cost are treated.
Indeed, in actual implementation, the distinctions between PERT and CPM have
become blurred as firms have integrated the best features of both systems into their own
efforts to manage projects effectively. The implementation of PERT and CPM had an
immediate impact on scheduling projects because it allowed the practice of “manage-
ment by exception.” Although there might be 10,000 activities in the course of a project,
perhaps only 150 of them would be “critical” and need to be watched closely. To put an
American on the moon during the days of the Apollo project, North American Aviation
used PERT to bring its part of the project in six weeks early. There were over 32,000
events and hundreds of thousands of activities, but only a few hundred needed constant
monitoring.

In keeping with our philosophy throughout the text, we approach the topic of project
management on two levels. First, the essential techniques will be developed in an easily
grasped illustrative example. Second, the use of the spreadsheet will be illustrated to indi-
cate how one would handle the techniques in a large-scale, real-world application.
A Typical Project: The Global Oil Credit Card Operation

No one would claim that it is like building the Great Pyramid, but the impending move of the credit card operation to Des Moines, Iowa, from the home office in Dallas is an important project for Rebecca Goldstein and Global Oil. The board of directors of Global has set a firm deadline of 22 weeks for the move to be accomplished. Becky is a manager in the Operations Analysis Group. She is in charge of planning the move, seeing that everything comes off according to plan, and making sure that the deadline is met.

The move is difficult to coordinate because it involves many different divisions within the company. Real estate must select one of three available office sites. Personnel has to determine which employees from Dallas will move, how many new employees to hire, and who will train them. The systems group and the treasurer’s office must organize and implement the operating procedures and the financial arrangements for the new operation. The architects will have to design the interior space and oversee needed structural improvements. Each of the sites that Global is considering is an existing building with the appropriate amount of open space. However, office partitions, computer facilities, furnishings, and so on, must all be provided.

A second complicating factor is that there is an interdependence of activities. In other words, some parts of the project cannot be started until other parts are completed. Consider two obvious examples: Global cannot construct the interior of an office before it has been designed. Neither can it hire new employees until it has determined its personnel requirements.

The Activity List

Becky knows that PERT and CPM are specifically designed for projects of this sort, and she wastes no time in getting started. The first step in the process is to define the activities in the project and to establish the proper precedence relationships. This is an important first step since errors or omissions at this stage can lead to a disastrously inaccurate schedule. Table 14.1 shows the first activity list that Becky prepares for the move (the columns labeled “Time” and “Resources” are indications of things to come). This is the most important part of any PERT or CPM project and usually is done with several people involved, so that no important activities are missed. This must be a group effort—not done in isolation.

Conceptually, Table 14.1 is straightforward. Each activity is placed on a separate line, and its immediate predecessors are recorded on the same line. The immediate predecessors

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>DESCRIPTION</th>
<th>IMMEDIATE PREDECESSORS</th>
<th>TIME</th>
<th>RESOURCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Select Office Site</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Create Organizational and Financial Plan</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Determine Personnel Requirements</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Design Facility</td>
<td>A, C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Construct Interior</td>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Select Personnel to Move</td>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Hire New Employees</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Move Records, Key Personnel, etc.</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Make Financial Arrangements with Institutions in Des Moines</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>Train New Personnel</td>
<td>H, E, G</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
sors of an activity are those activities that must be completed prior to the start of the activity in question. For example, in Table 14.1 we see that Global cannot start activity C, determine personnel requirements, until activity B, create the organizational and financial plan, is completed. Similarly, activity G, hire new employees, cannot begin until activity F, select the Global personnel that will move from Texas to Iowa, is completed. This activity, F, in turn, cannot start until activity C, determine personnel requirements, is completed.

The activity list with immediate predecessors and the yet-to-be-obtained time estimates will provide the essential ingredients to answer the first five questions at the start of this chapter. We shall shortly see how PERT and CPM are used to produce these answers. In practice, however, another graphical approach, the Gantt chart, also is used commonly to attack such problems. We thus make a slight detour to consider this precursor of the network approaches (PERT and CPM) before returning to the main thrust of the chapter.

**THE GANTT CHART**

The Gantt chart was developed by Henry L. Gantt in 1918 and remains a popular tool in production and project scheduling. Its simplicity and clear graphical display have established it as a useful device for simple scheduling problems. The Gantt chart for Becky’s problem is shown in Figure 14.1. Each activity is listed on the vertical axis. The horizontal axis is time, and the anticipated as well as actual duration of each activity is represented by a bar of the appropriate length. The chart also indicates the earliest possible starting time for each activity. For example, activity C cannot start before time 5 since, according to Table 14.1, activity B must be completed before activity C can begin. As each activity (or part thereof) is completed, the appropriate bar is shaded. At any point in time, then, it is clear which activities are on schedule and which are not. The Gantt chart in Figure 14.1 shows that as of week 13 activities D, E, and H are behind schedule, while G has actually been completed (because it is all shaded) and hence is ahead of schedule.

This simple example shows how the Gantt chart is mainly used as a record-keeping device for following the progression in time of the subtasks of a project. As Figure 14.1 shows, we can see which individual tasks are on or behind schedule. It seems important to note at this point that in the Gantt chart context the phrase “on schedule” means “it has been completed no later than the earliest possible completion time.” Thus Figure 14.1

![Figure 14.1](image-url)
shows that D and H could have been completed, at the earliest, by week 12. Since they are not completed by week 13 they are, in this sense, behind schedule. As we shall see, this is too simple a concept for whether an activity is on schedule. The appropriate point of view should be whether the overall project is being delayed in terms of a target completion date. The Gantt chart fails to reveal some of the important information needed to attack this question. For example, the Gantt chart fails to reveal which activities are immediate predecessors of other activities. In Figure 14.1 it may appear that F and I are immediate predecessors of G since G can start at 10 and F and I can each finish at 10. In fact, however, Table 14.1 tells us that only F is an immediate predecessor of G. A delay in I would not affect the potential starting time of G, or for that matter of any other activity. It is this type of “immediate predecessor” information that must be used to deduce the impact on completion time for the overall project. This latter type of information is of obvious importance to the manager. The overall weakness of Gantt charts is reflected by their uselessness in making such inferences. We shall now see that the network representation contains the immediate predecessor information that we need.

THE NETWORK DIAGRAM

In a PERT network diagram each activity is represented by an arrow that is called a branch or an arc. The beginning and end of each activity is indicated by a circle that is called a node. The term event is also used in connection with the nodes. An event represents the completion of the activities that lead into a node. Referring to the activity list in Table 14.1, we see that “select office site” is termed activity A. When this activity is completed, the event “office site selected” occurs.

Constructing the Network Diagram Figure 14.2 shows a network diagram for activities A through C. We emphasize at the outset that the numbers assigned to the nodes are arbitrary. They are simply used to identify events and do not imply anything about precedence relationships. Indeed, we shall renumber the node that terminates activity C several times as we develop the network diagram for this project, but correct precedence relationships will always be preserved. In the network diagram each activity must start at the node in which its immediate predecessors ended. For example, in Figure 14.2, activity C starts at node ➂ because its immediate predecessor, activity B, ended there. We see, however, that complications arise as we attempt to add activity D to the network diagram. Both A and C are immediate predecessors to D, and since we want to show any activity such as D only once in our diagram, nodes ➁ and ➃ in Figure 14.2 must be combined, and D should start from this new node. This is shown in Figure 14.3. Node ➄ now represents the event that both activities A and C have been completed. Note that activity E, which has only D as an immediate predecessor, can be added with no difficulty. However, as we attempt to add activity F, a new problem arises. Since F has C as an immediate predecessor, it would

![Network Diagram for Activities A through C](image-url)
emanate from node ③ (of Figure 14.3). We see, however, that this would imply that F also has A as an immediate predecessor, which is incorrect.

**The Use of Dummy Activities** This diagramming dilemma is solved by introducing a dummy activity, which is represented by a dashed line in the network diagram in Figure 14.4. This dummy activity is fictitious in the sense that it requires no time or resources. It merely provides a pedagogical device that enables us to draw a network representation that correctly maintains the appropriate precedence relationships. Thus, Figure 14.4 indicates that activity D can begin only after both activities A and C have been completed. Similarly, activity F can occur only after activity C is completed.

We can generalize the procedure of adding a dummy activity as follows. Suppose that we wish to add an activity A to the network starting at node N, but not all of the activities that enter node N are immediate predecessors of the activity. Create a new node M with a dummy activity running from node M to node N. Take those activities that are currently entering node N and that are immediate predecessors of activity A and reroute them to enter node M. Now make activity A start at node M. (Dummy activities can be avoided altogether if, instead of associating activities with arcs (commonly known as activity on the arc [AOA]), we associate them with nodes. An example of this activity on the node (AON) approach is presented in the box that follows. Furthermore, all of the discussion in Section 14.8 uses the AON approach).

Figure 14.5 shows the network diagram for the first activity list as presented in Figure 14.5. We note that activities G and H both start at node ⑥ and terminate at node ⑦. This does not present a problem in portraying the appropriate precedence relationships, since only activity J starts at node ⑥. This might, however, create a problem for certain software packages used to solve PERT and CPM problems. In some of these programs, each activity is identified by the number of its starting and ending node. If such a program is to be used, the representation of G and H in Figure 14.5 would lead the computer to regard them as the same activity. This would be incorrect, since in fact activities G and H are not the same.
A dummy activity can be used to cure this condition. Figure 14.6 illustrates the procedure. Since the dummy activity requires no time, the correct time and precedent relationships are maintained. This new representation has been introduced into Figure 14.7. Many software packages do not require that these dummy activities be input. Thus, for our purposes, they serve mainly the pedagogical goal of correctly portraying the precedence relations (i.e., as used in Figure 14.4).
The activity list and an appropriate network diagram are useful devices for representing the precedence relationships among the activities in a project. Recall that the board has set a firm goal of 22 weeks for the overall project to be completed. Before Becky can tell if she can meet this goal, she will have to incorporate time estimates into the process.

The PERT-CPM procedure requires management to produce an estimate of the expected time it will take to complete each activity on the activity list. Let us assume that Becky has worked with the appropriate departments at Global to arrive at the expected time estimates (in weeks) shown in Table 14.2. (In Section 14.4 we shall discuss in more detail the way in which these time estimates were produced.) Figure 14.7 shows the network diagram with the expected activity times appended in brackets.

### Table 14.2  First Activity List with Expected Activity Times in Weeks

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>DESCRIPTION</th>
<th>IMMEDIATE PREDECESSORS</th>
<th>EXPECTED ACTIVITY TIME</th>
<th>RESOURCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Select Office Site</td>
<td>—</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Create Organizational and Financial Plan</td>
<td>—</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Determine Personnel Requirements</td>
<td>B</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Design Facility</td>
<td>A, C</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Construct Interior</td>
<td>D</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Select Personnel to Move</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Hire New Employees</td>
<td>F</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Move Records, Key Personnel, etc.</td>
<td>F</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Make Financial Arrangements with Institutions in Des Moines</td>
<td>B</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>Train New Personnel</td>
<td>H, E, G</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
THE CRITICAL PATH CALCULATION

From Table 14.2 you can see (by adding up the separate expected activity times) that the total working time required to complete all the individual activities would be 39 weeks. However, the total calendar time required to complete the entire project can clearly be less than 39 weeks, for many activities can be performed simultaneously. For example, Figure 14.7 shows that activities A and B can be initiated at the same time. Activity A takes 3 weeks and B takes 5 weeks. If management arranges to begin both activities at the same time (at calendar time 0), both will be completed by calendar time = 5. To obtain a prediction of the minimum calendar time required for overall project duration, we must find what is referred to as a critical path in the network.

A path can be defined as a sequence of connected activities that leads from the starting node \( \text{\textcircled{1}} \) to the completion node \( \text{\textcircled{8}} \). For example, the sequence of activities B–I, requiring 10 weeks to complete, is a path. So is the sequence B–C–D–E–J, requiring 23 weeks to complete. You can identify several other paths in Figure 14.7. To complete the project, the activities on all paths must be completed. In this sense we might say that “all paths must be traversed.” Thus, we have just seen that our project will take at least 23 weeks to complete, for the path B–C–D–E–J must be traversed. However, numerous other paths must also be traversed, and some of these may require even more time. Our task will be to analyze the total amount of calendar time required for all paths to be traversed. Thus, we wish to determine the longest path from start to finish. This path, called the critical path, will determine the overall project duration, because no other path will be longer. If activities on the longest path are delayed, then, since these activities must be completed, the entire project will be delayed. For this reason the activities on the critical path are called the critical activities of the project. It is this subset of activities that must be kept on schedule.

This is the opposite problem from the one in Chapter 5 (the shortest-route problem). Here, the longest route from beginning (start) to end (finish) is needed. We can either change this PERT problem to a shortest-route model and use the algorithm in Chapter 5 or change the algorithm to fit the problem. It is easier in this case to change the algorithm.

Earliest Start and Earliest Finish Times Another difference between the shortest-route problem and this longest route (critical path) is that the interest is not just in the longest path in the network, but in the earliest and latest times each activity can be started and not affect the current solution. Thus, what is needed is sensitivity analysis of each activity and, therefore, finding the earliest and latest start times (and finish times) for each activity. We now specify the steps employed in finding a critical path. Fundamental in this process will be the earliest start time for each activity. To illustrate this idea, consider activity D, “design facility.” Now assume that the project starts at time zero and ask yourself: “What is the earliest time at which activity D can start?” Clearly, it cannot start until activity A is complete. It thus cannot start before time = 3. However, it also cannot start before the dummy activity (that requires 0 time) is complete. Since the dummy cannot start until B and C are complete (a total of 8 weeks), we see that D cannot start until 8 weeks have passed. In this calculation, it is crucial to note that activities A and B both start at time 0. After 3 weeks A is complete, but B still requires another 2 weeks. After a total of 5 weeks, B is complete and C can start. After another 3 weeks, a total of 8 from the start, C is completed. Thus, after 8 weeks, both A and C are complete and D can start. In other words,

earliest start time for activity D = 8 weeks

Another important concept is earliest finish time for each activity. If we let

\[ \text{ES} = \text{earliest start time for a given activity} \]
\[ \text{EF} = \text{earliest finish time for a given activity} \]
\[ t = \text{expected activity time for a given activity} \]
then, for a given activity, the relation between earliest start time and earliest finish time is

\[ EF = ES + t \]

For example, we have just shown that for activity D we have \( ES = 8 \). Thus, for activity D,

\[ EF = ES + t \]
\[ = 8 + 4 = 12 \]

We now recall that each activity begins at a node. We know that a given activity leaving a node cannot be started until \textit{all} activities leading into that node have been finished. This observation leads to the following rule.

\textbf{Earliest Start Time Rule:} The ES time for an activity leaving a particular node is the \textit{largest} of the EF times for all activities entering the node.

Let us apply this rule to nodes 1, 2, 3, and 4 of Becky’s network, Figure 14.7. The result is shown in Figure 14.8. We write in brackets the earliest start and earliest finish times for each activity next to the letter of the activity, as shown in Figure 14.8. Note that the earliest start time rule applied to activity D says that ES for activity D is equal to the larger value of the EF times for the two precedent activities C (via the dummy) and A. Thus, the ES for D is the larger of the two values \([8, 3]\), which is 8.

Continuing to each node in a \textit{forward pass} through the entire network, the values [ES, EF] are then computed for each activity. The result is shown in Figure 14.9. Note that the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig14.8}
\caption{Earliest Start Time Rule}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig14.9}
\caption{Global Oil Network with Earliest Start and Earliest Finish Times Shown}
\end{figure}
earliest finish time for J is 23 weeks. This means that the earliest completion time for the entire project is 23 weeks. This answers the first of the questions itemized in Section 14.1: “What is the expected project completion date?”

**Latest Start and Latest Finish Times** In order to identify possible start and completion dates, the activities on the critical path, and how long noncritical activities may be delayed without affecting the overall completion date (answering the third, fourth, and fifth questions of Section 14.1), we now proceed with a **backward pass** calculation. The idea is that since we now have a target completion date (23 weeks from the start of the project), we can work backward from this date, determining the latest date each activity can finish without delaying the entire project. The backward pass begins at the completion node, node ➈. We then trace back through the network computing what is termed a latest start time and latest finish time for each activity. In symbols,

\[
\begin{align*}
LS &= \text{latest start time for a particular activity} \\
LF &= \text{latest finish time for a particular activity}
\end{align*}
\]

The relation between these quantities is

\[LS = LF - t\]

For activity J we define the latest finish time to be the same as its earliest finish time, which is 23. Hence, for activity J,

\[LS = LF - t = 23 - 3 = 20\]

Since the latest start time for activity J is 20, the latest activities E, H, and G can finish is 20. Thus, the latest E can start is 20 – 8 = 12, the latest H can start is 20 – 2 = 18, and the latest G can start is 20 – 4 = 16. To determine the latest finish time for activity F is a little more complicated. We apply the following general rule:

**Latest Finish Time Rule:** The LF time for an activity entering a particular node is the smallest of the LS times for all activities leaving that node.

Thus, for activity F, which enters node ➇, we apply the rule to see that LF = 16, because the latest start times for the activities leaving node ➇ (activities H and G) are 18 and 16. The complete network with LS and LF entries is shown in Figure 14.10. These entries appear on the arc for each activity in brackets, directly under the ES and EF times.

![Diagram of the network showing LS and LF times](image_url)
Slack and the Critical Path  Based on Figure 14.10, the next step of the algorithm is to identify another important value, the amount of slack, or free time, associated with each activity. Slack is the amount of time an activity can be delayed without affecting the completion date for the overall project. Slack is the same concept covered in LP and is the extra time that could be spent on that path without affecting the length of the critical path. An easy and important mathematical check is that every activity on the critical path should have the same slack, namely 0. For each activity, the slack value is computed as

\[ \text{slack} = \text{LS} - \text{ES} = \text{LF} - \text{EF} \]

For example, the slack for activity G is given by

\[ \text{slack for G} = \text{LS for G} - \text{ES for G} \]
\[ = 16 - 10 \]
\[ = 6 \]

and the same value is given by

\[ \text{LF for G} - \text{EF for G} = 20 - 14 = 6 \]

This means that activity G could be delayed up to 6 weeks beyond its earliest start time without delaying the overall project. On the other hand, the slack associated with activity C is

\[ \text{slack for C} = \text{LS for C} - \text{ES for C} \]
\[ = 5 - 5 \]
\[ = 0 \]

Thus, activity C has no slack and must begin as scheduled at week 5. Since this activity cannot be delayed without affecting the entire project, it is a critical activity and is on the critical path.

PERT critical paths can also have alternative optimal solutions (again, just as in LP). If activity F took 8 weeks, then the path B-C-F-G-J would also be a critical path.

Spreadsheet Approach for the Network  The spreadsheet solution (GLOBAL.XLS) of this problem is most easily done with an activity-on-the-node approach (shown in the figure on page CD14-9) and is shown in Figure 14.11. The Earliest Finish column (E) is the Activity Time column (C) plus the Earliest Start column (D). Similarly, the Latest Start column (F) is the Latest Finish column (G) minus the Activity Time column (C). The Slack column (H) could either be the Latest Start column (F) minus the Earliest Start column (D) or equivalently the Latest Finish column (G) minus the Earliest Finish column (E). The formula for the minimum project length in cell E15 is

\[ =\text{MAX}(E2:E12), \text{the largest Earliest Finish time.} \]

The cumbersome part of creating the spreadsheet is entering the information that is conveyed by the network diagram. For example, since the Latest Finish time of activity F is the smallest of the Latest Start times of activities G, H, and K, the formula in cell G7 is =\text{MIN}(F8,F9,F12) as shown in Figure 14.11. Since the Earliest Start time of activity D is the largest of the Earliest Finish times of activities A and C, the formula in cell D5 is

\[ =\text{MAX}(E2:E4). \]

Note that the word “Yes” is printed in the “Critical?” column for those activities with zero slack. Thus, we can see from this spreadsheet output that the critical path for Becky’s project is B-C-D-E-J. The minimum overall completion time is 23 weeks, which is the sum of the times on the critical path, as well as the earliest finish time for the last activity (J). Figure 14.11 also provides the answers to questions 3, 4, and 5 raised in Section 14.1. In other words, we have, up to this point, answered the following questions from that section.
1. What is the expected project completion date?
   Answer: 23 weeks.

3. What are the scheduled start and completion dates for each specific activity?
   Answer: An activity may be scheduled to start at any date between “earliest start” and “latest start.” The scheduled completion date will be “start date + expected activity time.” For example, activity G can be scheduled to start anywhere between time = 10 and time = 16. As shown in Table 14.2, the expected activity time is 4 weeks. Hence, the scheduled completion date will be “start date + 4.”

4. What activities are critical in the sense that they must be completed exactly as scheduled in order to meet the target for overall project completion?
   Answer: The activities on the critical path: namely, B, C, D, E, J.

5. How long can noncritical activities be delayed before a delay in overall completion date is incurred?
   Answer: Any activity may be started as late as the “latest start” date without delaying the overall project completion.
Three questions, namely 2, 6, and 7, remain to be answered. But first, before proceeding further, let us take an overview of what we have learned. It is clear from the critical path analysis that Becky has a problem. The board of directors wants the credit card operation to start operating in Des Moines in 22 weeks, and with the current plan 23 weeks are required. Obviously, something must change if this goal is to be met.

WAYS OF REDUCING PROJECT DURATION

There are two basic approaches to reducing the time required to complete a project:

1. **A strategic analysis**: Here the analyst asks: “Does this project have to be done the way it is currently diagrammed?” In particular, “Do all of the activities on the critical path have to be done in the specified order?” Can we make arrangements to accomplish some of these activities in a different way not on the critical path?

2. **A tactical approach**: In this approach the analyst assumes that the current diagram is appropriate and works at reducing the time of certain activities on the critical path by devoting more resources to them. The current expected times assume a certain allocation of resources. For example, the 8 weeks for construction (activity E) assumes a regular 8-hour workday. The contractor can complete the job more rapidly by working overtime, but at increased costs.

The tactical approach will get us into consideration of CPM models, to be discussed in Section 14.6. For now, let us deal with the so-called strategic questions.

**A Strategic Analysis** Becky starts with a strategic analysis, since she is anxious to keep the cost of the move as low as possible. This is analogous to “What if?” analysis done with spreadsheets. After some study she suddenly realizes that the current network assumes that activity J, the training of new employees, must be carried out in the new building (after E is complete), and after records and key personnel have been moved (after H is complete). After reconsidering, she believes that these requirements can be changed. First of all, J can be accomplished independently of H. The previous specification that H should be an immediate predecessor of J was simply incorrect. Moreover, she believes that she can secure an alternative training facility by arranging to use surplus classroom space in Des Moines at a minimal cost. She can then have the new employees trained and ready to start the moment that construction ends. On the other hand, she has to add another activity to the activity list: Secure a training facility (to be denoted as activity K). Although she feels that such a rearrangement may be helpful, it is possible that in this redefined network she may have created a new critical path with a still unsatisfactory minimum time (i.e., one greater than 22 weeks).

**Spreadsheet Output for the Redefined Network** Figure 14.12 shows the redefined activity list in the form of the activity-on-the-arc (AOA) diagram. Note that the dummy activities are given a normal duration (expected activity time) of 0. Some of the entries (Crash duration, Normal cost, and Crash cost) do not have to be specified for this particular analysis. Becky has to do only a limited amount of modifying the input data from her previous definition of the network. Thus, she must add activity K and change the ending node numbers for activities E, G, and H. Figure 14.13 shows the network diagram for the redefined project in the activity-on-the-node (AON) diagram.

Becky then enters the information into the spreadsheet (“Redefined” worksheet in the same GLOBAL.XLS) to solve the problem, and the results shown in Figure 14.14 (page CD14-17) are produced. Here we see that the redefined project can be completed in 20 weeks (the sum of the times on the critical path), so the board’s deadline can be met. It is also apparent that the activity “train” (J) is no longer on the critical path. In spite of the fact that 3 weeks is needed to secure a training facility, activity J has a slack of 3 weeks. The spreadsheet solution shows that the critical path for the redefined project is B-C-D-E. (Recall that “Yes” in the “Critical?” column indicates that an activity is on the critical path.)
Let us now consider the second question raised in the introduction: “What is the potential variability in the expected project completion date?” So far, we have been acting as though the activity times and the derived values for ES, LS, EF, and LF were all deterministic. This may not be strictly correct, for in reality the activity times are often not known in advance with certainty. In view of this fact, PERT employs a special formula for estimating activity times. We shall now present the details, and in so doing it will be seen that the PERT approach can also be used to calculate the probability that the project will be completed by any particular time.

Estimating the Expected Activity Time

The PERT system of estimating activity times requires someone who understands the activity in question well enough to produce three estimates of the activity time:

1. **Optimistic time** (denoted by \( a \)): the minimum time. Everything has to go perfectly to achieve this time.
2. **Most probable time** (denoted by \( m \)): the most likely time. The time required under normal circumstances.
3. **Pessimistic time** (denoted by \( b \)): the maximum time. One version of Murphy’s Law is that if something can go wrong, it will. The pessimistic time is the time required when Murphy’s Law is in effect.
Consider, for example, activity E, construct the interior. Becky and the general contractor carefully examine each phase of the construction project and arrive at the following estimates:

\[
a = 4
\]
\[
m = 7
\]
\[
b = 16
\]

The relatively large value for \( b \) is caused by the possibility of a delay in the delivery of the air-conditioning unit for the computer. If this unit is delayed, the entire activity is delayed. Moreover, in this case, since E is on the critical path, a delay in this activity will delay overall project completion.

In the original development of the PERT approach (during the late 1950s), the procedure for estimating the expected value of the activity times was motivated by the assump-
tion that the activity time was a random variable with a particular probability distribution. This distribution (the beta distribution) has a minimum and maximum value, unlike the normal distribution, which has an infinite range of values. It also is capable of assuming a wide variety of shapes, again unlike the normal, which is always symmetrical about its most likely value. A typical beta distribution is shown in Figure 14.15. The expected value of a beta distribution is approximately \((a + 4m + b)/6\); thus the formula used to estimate the expected activity time is

\[
\text{estimated expected activity time} = \frac{a + 4m + b}{6} \quad (14.1)
\]

Note that the estimate is a weighted average of the values of \(a\), \(m\), and \(b\), where the weights \(\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\) sum to 1. This means that the estimate will always lie between \(a\) and \(b\). Thus for activity \(E\),

\[
\text{estimated expected activity time} = \frac{4 + 4(7) + 16}{6} = 8
\]

By working with the appropriate individuals in Global Oil, Becky used (14.1) to estimate each of the expected activity times that were presented in Table 14.2 and subsequently used in the critical path analysis.

**Estimating the Standard Deviation of an Activity Time** The standard deviation of an activity time is estimated by assuming that there are six standard deviations between the optimistic and pessimistic times:

\[
\text{estimated standard deviation of activity time} = \frac{b - a}{6} \quad (14.2)
\]

Thus, for activity \(E\),

\[
\text{estimated standard deviation} = \frac{16 - 4}{6} = 2
\]

Table 14.3 shows the three estimates \((a, m, b)\), the expected activity times, the standard deviation of the activity times, and the variance of the activity times for the redefined activity list. The variance is simply the square of the standard deviation. It is useful to record the variance of each activity since these values will be used in making statements about the probability of completing the overall project by a specific date.

In an application, it is of course possible to use any procedure that seems appropriate to estimate the expected value and standard deviation of the activity time. Indeed, in some circumstances data may be available and various statistical procedures can be used to estimate these parameters of the model.

**PROBABILITY OF COMPLETING THE PROJECT ON TIME**

The fact that activity times are random variables implies that the completion time for the project is also a random variable. That is, there is potential variability in the overall com-
pletion time. Even though the redefined project has an expected completion time of 20 weeks, there is no guarantee that it will actually be completed within 20 weeks. If by chance various activities take longer than their expected time, the project might not be completed within the desired 22-week schedule. In general, it would be useful to know the probability that the project will be completed within a specified time. In particular, Becky would like to know the probability that the move will be completed within 22 weeks.

The analysis proceeds as follows:

1. Let \( T \) equal the total time that will be taken by the activities on the critical path.
2. Find the probability that the value of \( T \) will turn out to be less than or equal to any specified value of interest. In particular, for Becky’s project we would find \( \text{Prob}\{T \leq 22\} \). A good approximation for this probability is easily found if two assumptions hold.
   a. *The activity times are independent random variables.* This is a valid assumption for most PERT networks and seems reasonable for Becky’s problem. There is no reason to believe that the time to construct the interior should depend on the design time, and so on.
   b. *The random variable \( T \) has an approximately normal distribution.* This assumption relies on the *central limit theorem*, which in broad terms states that the sum of independent random variables is approximately normally distributed.

Now recalling that our goal is to find \( \text{Prob}\{T \leq 22\} \), where \( T \) is the time along the critical path, we will want to convert \( T \) to a standard normal random variable and use Table A.0 in Appendix A to find \( \text{Prob}\{T \leq 22\} \). The first step in this process is to find the standard deviation of \( T \). To do this we need the variance of \( T \). When the activity times are independent, we know that the variance of the total time along the critical path equals the sum of the variances of the activity times on the critical path. Thus, for Becky’s problem

\[
\text{var} \ T = \left(\text{variance for activity B}\right) + \left(\text{variance for activity C}\right) + \left(\text{variance for activity D}\right) + \left(\text{variance for activity E}\right)
\]

Using the numerical values in Table 14.3 yields

\[
\text{var} \ T = 1 + \frac{1}{9} + \frac{4}{9} + 4 = \frac{50}{9}
\]

Finally,

\[
\text{std. dev.} \ T = \sqrt{\text{var} \ T} = \sqrt{\frac{50}{9}} = 2.357
\]
We now proceed to convert $T$ to a standard normal random variable, $Z$, in the usual way:

$$Z = \frac{T - \mu}{\sigma}.$$  

Recalling that 20 weeks is the mean (i.e., the expected completion time), we have

$$\text{Prob}(T \leq 22) = \text{Prob}\left(\frac{T - 20}{2.357} \leq \frac{22 - 20}{2.357}\right)$$

$$= \text{Prob}(Z \leq 0.8485)$$

If we consult Table A.0 at the end of the text for the area under a normal curve from the left-hand tail to a point that is 0.8485 standard deviations above the mean, we find that the answer is about 0.80. Thus, there is about an 80% chance that the critical path will be completed in less than 22 weeks.

This analysis shows how to shed light on the second of the questions asked in the introduction. In particular, it shows how to find the probability that the *critical path* will be finished by *any* given time. It illustrates the importance of considering the variability in individual activity times when considering overall project completion times. The analysis for Becky’s problem indicates that, using expected time as our “real-world forecast,” the expected project duration will be 20 weeks and, if so, it will be completed 2 weeks ahead of the desired date. The analysis of uncertainty above sheds additional light on this estimate. It shows a significant probability (i.e., 0.2 = 1 – 0.8) that the *critical path* will not be completed by the desired completion date. The implication is that there is at least a probability of 0.2 that the overall project may not be completed by the desired date. The modifier “at least” has been employed because of the following complicating factor: Because of randomness, some other path, estimated as being noncritical, may in reality take longer to complete than the purported critical path.

As an example of how this uncertainty can work in the business/educational world, San Diego State University and Georgia Tech contracted with some construction firms to build parking towers. The construction firms gave them two bids: one if PERT charts were used and a lower bid if PERT charts were *not* used. In the latter bids, the firms would not promise when the structures would be done so that if workers were needed on another project (which did have deadlines), they could be taken off the universities’ projects for several days or weeks and used elsewhere. In exchange for this uncertainty, the educational institutions got cheaper construction rates for helping the contractors with their personnel balancing.

**TESTING THE ASSUMPTIONS WITH SPREADSHEET SIMULATION**

For small projects, it is not too cumbersome to use a spreadsheet program to do a critical path analysis as we have shown previously with Figures 14.11 and 14.14. Once the basic relationships have been entered, it is very easy to alter the activity times and see what effect this has on the minimum project length and the activities on the critical path. By making the activity times random with Crystal Ball and recalculating the spreadsheet (using its “Single Step” feature), one can get a feel for the variability of both the project length and the critical path. Figure 14.16 shows one example (“Random” worksheet in the same GLOBAL.XLS). Note that all activity times are between their pessimistic ($a$) and optimistic ($b$) times, but that the critical path is different, in this case B-C-F-G-J. This result demonstrates that the path with the longest expected length (B-C-D-E) may not turn out to be the critical path. This fact implies that the expected project length may be greater than the value calculated by the PERT analysis.²

To estimate the true expected project length, we should recalculate the spreadsheet many times and average the minimum project lengths obtained on each recalculation. The

² The key result needed to show this is that the expected value of the maximum of two random variables is greater than or equal to the maximum of the expected values: $E[\max(X,Y)] \geq \max(E[X],E[Y])$. 
Crystal Ball add-in makes this easy to do. Since Crystal Ball includes the beta distribution in its Distribution Gallery, and the PERT analysis of activity times was based on the beta distribution, we indicate that the values in column C should be drawn from the appropriate beta distributions. Figure 14.17 shows the distribution of minimum project length calculated by Crystal Ball based on a sample of 1,000 iterations. The estimated average project length is in fact smaller than what was calculated by the PERT analysis, but only slightly so (19.76 versus 20.0). Crystal Ball also makes it easy to calculate the probability that the project length is less than or equal to any given target value. Figure 14.17 shows that for a target value of 22 weeks, that the probability is 83.6%, slightly higher than the 80% calculated by the PERT analysis. In this case, at least, it seems that PERT’s simplifying assumptions are justified.

Crystal Ball also makes it easy to generate histograms like the one shown in Figure 14.18. This is a histogram of the slack for activity F. While the expected slack is close to the
Using the PERT approach the analyst must provide the following inputs:

1. A list of the activities that make up the project.
2. The immediate predecessors for each activity.
3. The expected value for each activity time \[ t = (a + 4m + b)/6 \].
4. The standard deviation for each activity time \[ \text{std. dev. } t = (b - a)/6 \].

The PERT estimation procedure uses pessimistic, most likely, and optimistic estimates of the activity time to obtain the expected value and the standard deviation for each activity. The standard deviation is required only if the analyst wishes to make probability statements about completing the project by a certain date.

The analysis uses the inputs listed above to

1. Calculate the critical path;
2. Calculate the minimum expected time in which the project can be completed;
3. Show slack values for each activity, together with the latest expected time that any activity can start (or finish) without delaying the project; and
4. Calculate the probability that the current critical path will be completed by a specified date if estimates of the standard deviation are provided.

If the project cannot (or is unlikely to) be completed by a desired date, the project must be redefined either by

1. Strategic analysis, in which the project network is modified by introducing new activities or changing the relationships between existing activities, or
2. Tactical analysis, in which activity times are changed by the application of additional resources.

Finally, we can observe that PERT is not only a planning system. You can now see that it can also be used to monitor the progress of a project. Management can compare the actual activity times as they occur with those that were used in the planning process. If, for example, activity B took 6 or 7 weeks, rather than the 5 weeks used in the network diagram, Becky would know that the project is behind schedule. This would give her the opportunity to arrange to assign more resources to some other activity on the critical path in an effort to shorten that activity and meet the desired overall due date.

Identification of the critical path and prompt reporting give management a powerful tool to deal with the difficult problem of bringing a complicated project in on schedule.
time anywhere between a minimum value and a maximum value. The choice of an activity time implies an activity cost as specified by the diagram.

Given the availability of such a time-cost trade-off function for each activity in the project, management has the opportunity to select each activity time (within limits) and incur the associated cost. Clearly, the choice of individual activity times affects the project completion time. The question becomes: “What activity times should be selected to yield the desired project completion time at minimum cost?” The CPM approach to answering this question will be presented in the context of the creation of a financial analysis package by the Operations Analysis Group at Global.

**A FINANCIAL ANALYSIS PROJECT FOR RETAIL MARKETING**

In addition to the move to Des Moines, Becky is responsible for a new financial analysis package that will be used in the retail marketing section of Global. The program is used in evaluating potential outlets (gas stations) in terms of location and other characteristics. The systems design is complete. The computer programming must still be done, and the package must be introduced to the retail marketing section.

Figure 14.20 shows the activity list and network diagram for this project. The time shown is termed the **normal time**. This corresponds to the maximum time shown in Figure 14.19. Recall that we are here assuming that activity times can be estimated with good accuracy, and hence “normal time” is a known quantity. From Figure 14.20 it is clear that the longest path through the network is DAP-WAP-INT, and hence this is the critical path. The earliest completion time for the project is 194 hours.

**Required Activity Data** The CPM system is based on four pieces of input data for each activity:

1. **Normal time**: the maximum time for the activity.
2. **Normal cost**: the cost required to achieve the normal time.
3. **Crash time**: the minimum time for the activity.
4. **Crash cost**: the cost required to achieve the crash time.
These data for the financial analysis project are presented in the first four columns of Table 14.4. The fifth column shows the maximum crash hours, defined by

\[
\text{Max crash hours} = \text{normal time} - \text{crash time}
\]

Figure 14.21 shows how these data are used to create the time-cost trade-off function for activity DIP, design the information processor.

Note that, according to Table 14.4, using all normal times leads to a total project cost of $7,208. Also note that the last column in Table 14.4 shows how much it costs per hour (as computed in Figure 14.21) to reduce each activity time to less than its normal time. In CPM jargon, the process of reducing an activity time is called crashing. For example, management could choose to have DIP completed in 31 hours, rather than the normal 32 hours, for a marginal cost of $13.33. The normal time of 32 hours costs $640, and a time of 31 hours would therefore cost $640 + 13.33 = $653.33.

### CRASHING THE PROJECT

We have noted that, using only the normal time for each activity, the earliest completion time for this project is 194 hours (along the critical path DAP-WAP-INT). Management is now in a position to determine the minimum-cost method of reducing this time to specified levels. To reduce the project time to 193, Becky would crash an activity on the critical path by 1 hour. Since it costs less per hour to crash WAP than either of the other two activities on the critical path ($8 < $10 and $8 < $16), Becky would first crash WAP by 1 hour. This decision yields a project time of 193 hours, a critical path of DAP-WAP-INT, and a total project cost of $7,216 (= $7,208 + $8). If Becky wants to achieve a time of 192 hours, exactly the same analysis would apply, and she would crash WAP by another hour and incur a marginal cost of $8.

If Becky has crashed WAP by 2 hours to achieve a project time of 192 hours and still wants to crash the project by another hour (to achieve 191), the analysis becomes more complex, requiring a more detailed calculation of the cost savings and time benefits of crashing additional activities.

### Table 14.4  Time-Cost Data for the Financial Analysis Project

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>(1) NORMAL TIME</th>
<th>(2) NORMAL COST ($)</th>
<th>(3) CRASH TIME</th>
<th>(4) CRASH COST ($)</th>
<th>(5) MAXIMUM CRASH HOURS</th>
<th>(6) COST PER CRASH HOUR ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIP</td>
<td>32</td>
<td>640</td>
<td>20</td>
<td>800</td>
<td>12</td>
<td>13.33</td>
</tr>
<tr>
<td>WIP</td>
<td>40</td>
<td>480</td>
<td>30</td>
<td>720</td>
<td>10</td>
<td>24.00</td>
</tr>
<tr>
<td>DAP</td>
<td>50</td>
<td>1000</td>
<td>30</td>
<td>1200</td>
<td>20</td>
<td>10.00</td>
</tr>
<tr>
<td>WAP</td>
<td>24</td>
<td>288</td>
<td>15</td>
<td>360</td>
<td>9</td>
<td>8.00</td>
</tr>
<tr>
<td>INT</td>
<td>120</td>
<td>4800</td>
<td>70</td>
<td>5600</td>
<td>50</td>
<td>16.00</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$7,208</td>
</tr>
</tbody>
</table>

**FIGURE 14.21**

Time-Cost Trade-off Function for DIP

Total cost

Crash time, crash cost

Slope = \(-13.33 = \frac{800 - 640}{12}\)

Normal time, normal cost

20 32 Time (hours)
complicated. Figure 14.22 shows the situation. The dollar figure in the diagram is the marginal cost of crashing. Note that there are now two critical paths, DIP-WIP-INT and DAP-WAP-INT, and that both require 192 hours. Crashing one of the four activities (DIP, WIP, DAP, or WAP) by 1 hour would bring one path down to 191 hours, but would still leave the project time at 192, since there would still be a critical path of 192 hours. A time of 191 could be obtained only by crashing activities on both paths. If Becky crashed DIP and WAP by 1 hour each it would reduce both paths to 191 hours, and it would cost her $13.33 + $8.00 = $21.33. Alternatively, INT could be crashed by 1 hour for a cost of $16.00. Can you see that there are other alternatives to consider?

Although it is possible to do this sort of marginal cost analysis in any CPM network, it is clear that it would be difficult and tedious to carry it out in a complicated network. This consideration leads us to an LP formulation of the problem.

A LINEAR PROGRAMMING MODEL

The problem of obtaining a specific project time at minimum cost can be formulated as a linear programming problem. Figure 14.23 shows the spreadsheet formulation and solution of the financial analysis problem (GLOBFIN.XLS) with a limit of 184 hours on the project time. To understand this formulation, let

- \( CWIP = \) hours crashed on activity WIP
- \( ESWIP = \) earliest start time for activity WIP
- \( EFINT = \) earliest finish time for activity INT

![FIGURE 14.22](image)

![FIGURE 14.23](image)
The other decision variables follow this same pattern. From these definitions it follows that

1. The objective function is the total cost of crashing the network. This is the appropriate objective. The cost of completing the project on normal time is already determined. You can think of management’s problem as deciding how much (and where) to crash to obtain the desired earliest finish time at minimum additional cost.

2. The constraints in cells B9:D12 establish limits for the earliest start time for activities WIP, WAP, and INT. For example, rewriting the first constraint (ESWIP), whose formula is in B9:D9, yields

   $$ESWIP \geq 32 - CDIP$$

   We note that since 32 is the normal time for DIP and CDIP is the amount that DIP is crashed, the right-hand side is the time that activity DIP will take after it has been crashed. Thus, this constraint states that the earliest start time for WIP must be ≥ the modified activity time for DIP. Since DIP is the only predecessor for WIP, we know that the earliest start time for WIP is exactly this modified activity time of DIP. You will thus expect to see

   $$ESWIP = 32 - CDIP$$

   A ≥ constraint is used rather than an = constraint because, in general, there could be several paths leading into a node, and the earliest start time of an activity leaving that node is determined by the entering path that takes the longest time. This is illustrated by the two constraints for ESINT. Rewriting these constraints yields

   cells B11:D11 → ESINT ≥ 40 – CWIP + ESWIP
   cells B12:D12 → ESINT ≥ 24 – CWAP + ESWAP

   The first constraint states that INT cannot start until WIP is complete, and the second makes a similar statement for WAP. It will often be the case that only one of these will be active in an optimal solution.

3. Cells B13:D13 contain the definition of earliest finish time for activity INT:

   $$EFINT = ESINT + 120 - CINT$$

   activity time for INT after crashing

4. The last constraint (cells B14:D14) sets an upper limit on the project time we want to achieve. This constraint depends on the fact that the finish time for activity INT determines the finish time for the overall project. In general, a similar constraint would be required for each activity leading into the terminal mode. Here we have only one such activity, INT.

5. Finally we need to add some constraints directly in the Solver Dialog box that limit the amount of crashing on each activity. The limit is given originally by column 5 in Table 14.4 “Maximum Crash Hours.” For the Solver, we simply indicate that


   The solution in Figure 14.23 shows that WAP should be crashed by 2 hours and INT by 8 hours to achieve a minimum cost reduction of 10 hours in the project completion time (i.e., the optimal value of EFINT is 184, and this is the project completion time). As usual with LP output, however, this is only a small part of the information available. For example, Figure 14.24 shows us that it will cost $16 to crash the network for 1 additional hour (shadow price on “Early Finish INT Goal”). The right-hand-side ranges show that this rate of $16 per hour holds for a decrease of another 42 hours. In this simple problem you can see that this next 42 hours of crashing (beyond the first 10) should be done on INT. In general this type of LP sensitivity information can provide useful guidance to management in the attempt to control the progress of large projects.
On February 27, 1991, President Bush stated, “We said the aggression would not stand, and the aggression did not stand!” The armed conflict in the Kuwaiti Theater of Operations ended in one of the most resounding military victories in history. How was project management instrumental in the successful execution of the military operation and particularly in those fateful 100 hours of the ground war?

General Schwarzkopf directed that the logistics community keep a 60-day supply of material on hand. This included medical supplies, diesel fuel, motor gasoline, and so forth. Looking at medical supplies alone, the best initial estimate was 26,352 short tons. Then the figure was modified based on the specifics of operations in Kuwait. For example, the amount of saline solution for rehydration of personnel was greater than that required in Europe (due to the arid climate). Also, there were insufficient refrigeration units on hand to properly store whole blood. U.S. ground forces were projected to use 1,600,000 gallons of diesel fuel and 180,000 gallons of motor gasoline per day, but when the forces went on the offensive those figures would double. The combat units usually carry only one to two days’ worth of supplies, and thus rely heavily on a steady source of new supplies. The combat unit is specifically designed for mobility and survivability on the battlefield, not for storing extraneous amounts of supplies.

One of the most successful results was a resupply technique that cut the resupply time for a heavy brigade in half (from six hours to three hours), and allowed both the supply unit and supported combat unit to be moving during resupply activities. Minimizing the resupply time has three major benefits. First, it increases security. While the vehicles are congested for resupply, they are more subject to detection by the enemy. Second, the supply vehicles can make more round trips in the same amount of time. Third, the tactical commander has greater flexibility in planning because less time is taken up by resupply activities.

A famous saying in the U.S. forces is “tacticians win battles, but logistics win wars.” All in all, the successful implementation of the numerous details of such a complex project as Desert Storm is a large part of what made the victory so decisive. (See Staats.)
In concluding this section, we recall the sixth question raised in Section 14.1: “How might resources be concentrated most effectively on activities in order to speed up project completion?” In this section we have seen that in a context where time and costs are suitably defined, as in the CPM model, project crashing allows management to answer this question.

We now proceed to discuss the final question raised in Section 14.1: “What controls can be exercised on the flows of expenditures for the various activities throughout the duration of the project in order that the overall budget can be adhered to?”

The desirability of a project typically depends on its total costs and revenues. (Discounting to account for the time value of money may be necessary to express costs and/or returns in current dollars if the project is of long duration.) Once a project has been selected, effective cost management includes two important functions: planning and control.

**PLANNING COSTS FOR THE CREDIT CARD PROJECT: THE PERT/COST SYSTEM**

Large projects can strongly influence the financial situation within a firm. The need to pay for the various activities creates a demand on both the firm’s overall budget and the daily cash flow. Obviously, the times at which activities are scheduled determine when budget demands occur. This is a good example of how management science models can be hooked together. First, a PERT network model is made of the Global Oil move, then perhaps an LP model is used to crash to an acceptable time frame, and finally a spreadsheet financial model can be made to aid in financial planning. Thus, the output of each model becomes the input for the next, and so it is crucial to constantly check the model results for accuracy.

It is important for a firm to be able to anticipate budget demands in order to be able to handle them economically and effectively. The PERT/Cost system is specifically designed to help management anticipate such demands in a clear and consistent manner. PERT/Cost is essentially an alternative approach to cost accounting. Typically, cost accounting systems are organized on a cost center basis (e.g., by departments). The PERT/Cost system is organized on a project basis, where the basic elements of control are the activities.

In order to apply the PERT/Cost system to the project of moving the credit card operation to Des Moines, Becky must now complete Table 14.2 by filling in the final column, titled “Resources.” This is an estimated or “expected” total cost of completing each activity. These expected activity costs, together with the expected activity times, the earliest start time, and the latest start time, are presented in Table 14.5 for the redefined credit card project. The earliest start and latest start data are taken from the spreadsheet solution, Figure 14.14.

The goal of the PERT/Cost system is to construct a graph of budget demands over time. This requires knowledge of how funds will be spent throughout the life of an activity. For example, the demands on the budget are different if the $32,000 for activity E, construct the interior, is due at the beginning of the 8-week activity time or at the end of it. PERT/Cost makes the assumption that expenditures occur uniformly throughout the life of the activity; that is, for E a budget demand of $4,000 occurs during each of the 8 weeks. Table 14.6 shows the budget demands by time if all activities start at their earliest start time. This table is constructed by assigning a row to each activity and recording the budget demands for that activity in the appropriate column (week) as determined by the earliest start time. In forming this table, “earliest start” times are interpreted as referring to the end of the appropriate week. Thus activity B starts at time 0 (the end of week 0 = beginning of week 1) and requires 5 weeks to complete. This means that activity C, as shown in Table 14.6, cannot start until the end of week 5. It lasts 3 weeks and makes a budget demand of $600 per week. This information is summarized in the third row of Table 14.6.
The total weekly cost is determined by adding down a column, that is, by adding the budget demands during the week from all the activities. For example, the budget demand during the thirteenth week is $5,200, the sum of $4,000 from E, $700 from G, and $500 from K.

The cumulative project cost is found by cumulating the weekly costs from the beginning of the project. For example, note that the weekly cost is $1,700 for each of the first 3 weeks. The total project cost after 3 weeks is therefore $5,100. The total cost at the end of the project (week 20) must, of course, be the total cost for the entire project.

Table 14.7 creates the profile of budget demands over time if each activity starts at its latest start time.

The information from Tables 14.6 and 14.7 is combined in Figure 14.25. The upper line is a plot of the earliest start time costs from Table 14.6, and the lower line is a plot of the latest start time costs from Table 14.7. The shaded area between the lines shows the area of feasible cumulative budgets for total project costs if the project is completed on time. The fact that the actual budget demands must fall within the envelope created by the earliest start time and the latest start time makes it easy for management to anticipate its cumulative expenditures. For example, Becky can see that by the end of week 12 Global Oil will have to have spent between $14,200 and $28,100.

We have progressed step by step through the budget calculations for the PERT/Cost planning system because this is a useful exercise from the pedagogical point of view. In practice these calculations are typically done on a computer. Figure 14.26 shows a computer output that corresponds to Tables 14.6 and 14.7.

### Controlling Project Costs

The concept behind any control system is to compare the actual performance with planned performance and to take remedial action if it is necessary. The thermostat in your house is a control system that operates continuously in time by comparing the actual temperature with the desired temperature and turning the furnace (air conditioner) off or on as necessary.

The PERT/Cost system compares actual costs or budgeted project costs at regular intervals so that management has an early indication if the project is not proceeding according to plan. Management is then in a position to take appropriate action.
| ACTIVITY | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A        | 700 | 700 | 700 | 700 | 1000| 600 | 600 | 600 | 1200| 1200| 1200| 1200| 1200| 1200| 4000| 4000| 4000| 4000| 4000| 4000|
| B        | 1000| 1000| 1000| 1000| 1000| 500 | 500 | 500 | 700 | 700 | 700 | 700 | 700 | 700 | 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000|
| C        | 600 | 600 | 600 | 1200| 1200| 700 | 700 | 700 | 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000|
| D        | 1200| 1200| 1200| 1200| 1200| 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 |
| E        | 1200| 1200| 1200| 1200| 1200| 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 |
| F        | 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000| 4000|
| G        | 800 | 800 | 800 | 500 | 500 | 500 | 500 | 500 | 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000| 10,000|
| H        | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |
| I        | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |
| J        | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |
| K        | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |

**Weekly Cost**

|       | 1700 | 1700 | 1700 | 1000 | 1000 | 1400 | 1400 | 1400 | 2500 | 2500 | 5900 | 5900 | 5200 | 4700 | 14,000| 14,000| 14,000| 14,000| 14,000| 14,000| 14,000| 14,000| 14,000| 14,000|
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|

**Cumulative Project Cost**

|       | 1700 | 3400 | 5100 | 6100 | 7100 | 8500 | 9900 | 11,300| 13,800| 16,300| 22,200| 28,100| 33,500| 38,000| 52,000| 66,000| 80,000| 84,000| 88,000| 92,000|
| ACTIVITY | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A        | 1000 | 1000 | 1000 | 1000 | 700 | 700 | 700 | 600 | 600 | 600 | 1200 | 1200 | 1200 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 |
| B        | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 800 | 800 | 800 | 10,000 | 10,000 | 10,000 |
| C        | 1000 | 1000 | 1000 | 1000 | 600 | 600 | 600 | 700 | 700 | 700 | 700 | 700 | 700 | 800 | 800 | 800 | 10,000 | 10,000 | 10,000 |
| D        | 1000 | 1000 | 1000 | 1000 | 600 | 600 | 600 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 |
| E        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 |
| F        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 |
| G        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 |
| H        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 |
| I        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 |
| J        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 |
| K        | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 | 1700 |
| **Weekly** | **1000** | **1000** | **1000** | **1000** | **1300** | **1300** | **1300** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1200** | **1300** |
| **Cost**  | **1000** | **2000** | **3000** | **4000** | **5000** | **6300** | **7600** | **8900** | **10,100** | **11,300** | **12,500** | **14,200** | **18,700** | **23,400** | **28,600** | **34,600** | **40,600** | **55,400** | **73,700** | **92,000** |
| **Cumulative Project Cost** | **1000** | **2000** | **3000** | **4000** | **5000** | **6300** | **7600** | **8900** | **10,100** | **11,300** | **12,500** | **14,200** | **18,700** | **23,400** | **28,600** | **34,600** | **40,600** | **55,400** | **73,700** | **92,000** |
FIGURE 14.25
Cumulative Budget Demands Versus Time

FIGURE 14.26
Computer Analysis of Budget Demands

RESOURCE USAGE PROFILE ACCORDING TO EARLIEST START TIME

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Resource Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>From End of Week</td>
<td>To End of Week</td>
</tr>
<tr>
<td>0.00→</td>
<td>3.00</td>
</tr>
<tr>
<td>3.00→</td>
<td>5.00</td>
</tr>
<tr>
<td>5.00→</td>
<td>8.00</td>
</tr>
<tr>
<td>8.00→</td>
<td>10.00</td>
</tr>
<tr>
<td>10.00→</td>
<td>12.00</td>
</tr>
<tr>
<td>12.00→</td>
<td>13.00</td>
</tr>
<tr>
<td>13.00→</td>
<td>14.00</td>
</tr>
<tr>
<td>14.00→</td>
<td>17.00</td>
</tr>
<tr>
<td>17.00→</td>
<td>20.00</td>
</tr>
</tbody>
</table>

RESOURCE USAGE PROFILE ACCORDING TO LATEST START TIME

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Resource Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>From End of Week</td>
<td>To End of Week</td>
</tr>
<tr>
<td>0.00→</td>
<td>5.00</td>
</tr>
<tr>
<td>5.00→</td>
<td>8.00</td>
</tr>
<tr>
<td>8.00→</td>
<td>11.00</td>
</tr>
<tr>
<td>11.00→</td>
<td>12.00</td>
</tr>
<tr>
<td>12.00→</td>
<td>13.00</td>
</tr>
<tr>
<td>13.00→</td>
<td>14.00</td>
</tr>
<tr>
<td>14.00→</td>
<td>15.00</td>
</tr>
<tr>
<td>15.00→</td>
<td>17.00</td>
</tr>
<tr>
<td>17.00→</td>
<td>18.00</td>
</tr>
<tr>
<td>18.00→</td>
<td>20.00</td>
</tr>
</tbody>
</table>
The PERT/Cost Control Report  Table 14.8 is a PERT/Cost control report prepared 11 weeks after the start of the redefined project to move the credit card operation to Des Moines. The labels on the columns indicate how the report is prepared. Column (4), the actual cost, and column (3), the budgeted cost, provide the basic information used in the control function. The actual cost, column (4), is self-explanatory. The budgeted cost, column (3), is calculated on the assumption that the percentage of budget used up by an activity is the same as the percentage of that activity that is completed. Thus, when an activity is 50% complete, its budgeted cost is 50% of the entire budget (column [2]) for that activity. Note that if an activity is completed, the entry in column (1) is 100, and this means the entry in column (3) will be the same as the entry in column (2).

Consider, for example, activity A, “select office site.” We see from column (1) of Table 14.8 that this activity, by the end of week 11, is 100% complete. Thus, its budgeted cost, column (3), is equal to its entire budget (column [2]) of $2,100. Since its actual cost is $2,300, there is a cost overrun of $200. This number is recorded in the last column. A similar interpretation applies to activity 1, “make financial arrangements.” It is 20% complete and has a budget of $4,000; thus, its budgeted cost is $800 ($800 = 0.20 \times 4000). Since only $500 has been spent, there is a budget surplus of $300. The parentheses in the last column indicate a budget surplus.

In this situation, since activities A and F are already completed, their cost overruns cannot be corrected. However, activities D, G, and H, none of which are yet complete, are showing significant overruns to date, and these activities should be promptly reviewed. This type of periodic managerial intervention is often required to keep the total project cost within the budget.

Potential Implementation Problems  Although PERT/Cost can provide an effective control procedure, it is well to be aware of potential implementation problems. For example, the required recording of data can involve significant clerical effort, especially when many projects with many activities are under way. Moreover, some costs, such as overhead, may be common to several activities. The allocation of such common costs can be problematic. Finally, as we mentioned at the beginning of Section 14.7, the PERT/Cost system differs in organization from typical cost accounting systems. The typical departmental cost center orientation needs to be substantially revised to handle the PERT/Cost activity-oriented system. Such redesign may be politically as well as materially expensive.

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>PERCENT COMPLETE</th>
<th>BUDGET ($)</th>
<th>[1/100 \times (2)]</th>
<th>BUDGETED COST TO DATE ($)</th>
<th>ACTUAL COST TO DATE ($)</th>
<th>COST OVERRUN ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>2100</td>
<td>2100</td>
<td>2300</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>5000</td>
<td>5000</td>
<td>4900</td>
<td>(100)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>75</td>
<td>4800</td>
<td>3600</td>
<td>4600</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>32,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>100</td>
<td>1000</td>
<td>1000</td>
<td>1200</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>25</td>
<td>2800</td>
<td>700</td>
<td>1400</td>
<td>700</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>50</td>
<td>7000</td>
<td>3500</td>
<td>5400</td>
<td>1900</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>20</td>
<td>4000</td>
<td>800</td>
<td>500</td>
<td>(300)</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>0</td>
<td>30,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0</td>
<td>1500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>92,000</td>
<td>18,500</td>
<td>22,100</td>
<td>3,600</td>
<td></td>
</tr>
</tbody>
</table>
Specialty software, such as Microsoft Project 98, makes the task of project management easier than doing it in a spreadsheet or by hand. In particular, this software greatly simplifies the processes of generating the links between tasks and giving us a graphical view of the critical activities and the whole project. Unfortunately, such software cannot do the optimization as shown in Section 14.6 and it requires an additional add-in (e.g., @Risk for Project) to do any sort of simulation of results on random completion times as shown in Section 14.4. In this section we present the same example used in Section 14.2 in order to show how to use Microsoft Project 98. You may want to check out the enclosed student CD or the web site www.microsoft.com/project for free trial software and some sample tutorials. In spite of the minor drawbacks mentioned above, most projects are managed with the help of software like MS Project, rather than spreadsheets that have larger drawbacks.

A PROJECT MANAGEMENT EXAMPLE: GLOBAL OIL CREDIT CARD OPERATION

As initially outlined in Section 14.2 (and briefly reviewed here), the impending move of the credit card operation to Des Moines, Iowa, from the home office in Dallas is an important project for Rebecca Goldstein and Global Oil. The board of directors of Global has set a firm deadline of 22 weeks for the move to be accomplished.

The first step in the process is to define the activities in the project and to establish the proper precedence relationships. Table 14.1 shows the first activity list that Becky prepares for the move. To enter these activities in MS Project, you must first make sure that you have chosen the Gantt Chart “view” (View Gantt Chart from the main menu) and then simply enter the ten activities and their expected duration as shown in Figure 14.27. Note: MS Project consistently refers to these activities as “tasks” so we’ll use those two words interchangeably. We also arbitrarily set the start date for tasks 1 and 2 to be 9/4/00. The next step is to create the “links” (as MS Project calls them) between the activities or to establish the precedence relationship between the activities as we discussed in Section 14.2. For example, activity G, hire new employees, cannot begin until activity F, select the personnel that will move from Texas to Iowa, is completed. We said earlier that F is an immediate predecessor of activity G. MS Project calls this a “Finish-Start” link, meaning activity F must finish before activity G can start. To enter this in MS Project, we first click on activity F (Select Personnel to Move), then hold the “Ctrl” key down on the keyboard and click on activity G (Hire New Employees), then click on the “Link” icon of the main menu as shown in Figure 14.28. Note: MS Project numbers the activities 1–10 rather than using the letter designations we have used (A–J). The order in which you choose the activities is very important! If we would have selected activity G first and then activity F, MS Project would have assumed...
In order to accomplish this linking with multiple predecessors, you must enter each relationship separately. For example, both activities A and C must be done before activity D can commence. To enter this in MS Project, we first click on activity A (Select Office Site), then hold the “Ctrl” key down on the keyboard and click on activity D (Design Facility), then click on the “Link” icon of the main menu. Next we click on activity C (Determine Personnel Requirements), then hold the “Ctrl” key down on the keyboard and click on activity D (Design Facility), then click on the “Link” icon. This sequence tells MS Project that both activities A and C must be completed before activity D can start. When we are done entering all the predecessors of Table 14.1, the final result is shown in Figure 14.29 (only displayed for activities C–I or what MS Project calls tasks 3–9). Once this information is entered, MS Project automatically does the forward and backward passes discussed in Section 14.3 and can display many helpful items of information for us. For example, to see a network diagram, we simply click on View, PERT Chart and MS Project displays the diagram shown in Figure 14.30. Notice that the critical path (B-C-D-E-J or 2-3-4-5-10) is displayed in red on the screen (versus black for the noncritical tasks). Finally, to display the slack for each of the activities, click on View, More Views . . . , Detail Gantt, Apply and we see on the Gantt chart that the critical activities are shown in red, noncritical activities are
shown in blue, and then with thin-blue lines (as opposed to the thicker lines for the activity times) the slack time is indicated visually with the actual value written out at the end of the arrow as shown in Figure 14.31 for activities G (6 weeks) and H (8 weeks) (or for tasks 7 and 8 as MS Project calls them). Note how much easier it is to do project management with such specialty software as MS Project! In just in few short steps we’ve covered the same ground as Sections 14.2 and 14.3. The completed MS Project file is saved on the student disk as GLOBAL.MPP.

As far as covering the topic in Section 14.4 (variability in activity times), as mentioned earlier, MS Project requires an additional add-in (e.g., @Risk for Project) to do any sort of simulation of results on random completion times. The best we can do without @Risk is to enter optimistic times, most probable times, and pessimistic times for each activity. Then MS Project will display for us an optimistic Gantt chart, a most likely Gantt chart, and a pessimistic Gantt chart. We will demonstrate this with the revised eleven activity Global Oil project (the revision described at the end of Section 14.3 in order to get the total project length down to 20 weeks).

To enter the range of times (optimistic, most probable, pessimistic) for each activity in MS Project, we select View, Toolbars>, PERT Analysis, which adds a new toolbar to the menu. From the new toolbar, we select the “PERT Entry Sheet” icon (the last of the seven new ones displayed), and enter the information as displayed in the chart shown in Figure 14.32. Once this data is entered, we can choose to look at any of the three available Gantt charts (optimistic, most probable, pessimistic) by clicking on one of the three new icons for this purpose on the new toolbar (the first three shown). As an example, the pessimistic Gantt chart is displayed in Figure 14.33 (for activities C–J or tasks 3–10) along with the
FIGURE 14.31
Slack Times for Activities G and H

FIGURE 14.32
PERT Entry Sheet for Revised Global Project

FIGURE 14.33
Pessimistic Gantt Chart for Revised Global Project
icon that caused it to be displayed. Here we see that the expected completion time for the whole project has grown from 100 days (20 weeks) to 175 days (35 weeks)!

As far as coverage of the topic in Section 14.6 (Time-Cost Trade-offs), MS Project cannot do linear programming optimization to determine what would be the best way to reduce the total project time, but you can add additional "resources" to each activity to reduce the time it would take. For example, going back to the base case for the normal completion times, activity A had an expected completion time of 3 weeks and MS Project assumes that is with 100% effort (or 1 person). We can decide to expend 200% effort on this task (i.e., effectively putting 2 people on the task) and MS Project will then reduce the expected completion time to 1.5 weeks. To do this in the software, we first must create a resource that corresponds with each activity and then link the resources with the activities.

To do this in MS Project, we simply do the following procedure:

1. On the View menu, click on Resource Sheet.
2. On the View menu, point to Table:, and then click Entry.
3. In the "Resource Name" field, type a resource name in a blank field (e.g., "Select Site" is the resource name we created that we’ll link up with the task "Select Office Site" in the next paragraph).
4. Steps 1–3 are repeated for all eleven tasks.

The other option that MS Project offers for creating resources is to list individual workers by name that are available to work on the tasks. The next step is to make the connection for the software between the eleven resources we just created and the eleven activities. This is done rather easily by following these steps:

1. Select View, then Gantt Chart.
2. In the “Task Name” column, select the task to which you want to assign a resource (e.g., Select Office Site) by clicking on it with the mouse.
3. Click on the “Assign Resources” icon, which brings up an “Assign Resources” table.
4. In the “Name” column, select the resource you want to assign to the task (e.g., Select Site).
5. Click Assign (a check mark to the left of the Name field indicates that the resource is assigned to the selected task).
6. Repeat steps 1–5 for all eleven tasks.

At the completion of these steps, you should have a list of tasks and resources that looks something like the following:

<table>
<thead>
<tr>
<th>Task/Activity</th>
<th>Corresponding Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select Office Site</td>
<td>Select Site</td>
</tr>
<tr>
<td>Create Organizational and Financial Plan</td>
<td>Create Plan</td>
</tr>
<tr>
<td>Determine Personnel Requirements</td>
<td>Personnel Reqmts</td>
</tr>
<tr>
<td>Design Facility</td>
<td>Design Facility</td>
</tr>
<tr>
<td>Construct Interior</td>
<td>Construct</td>
</tr>
<tr>
<td>Select Personnel to Move</td>
<td>Select Personnel</td>
</tr>
<tr>
<td>Hire New Employees</td>
<td>Hire New</td>
</tr>
<tr>
<td>Move Records, Key Personnel, etc.</td>
<td>Move Records</td>
</tr>
<tr>
<td>Make Financial Arrangements with Institutions in Des Moines</td>
<td>Make Financial</td>
</tr>
<tr>
<td>Train New Personnel</td>
<td>Train</td>
</tr>
<tr>
<td>Secure Training Facility</td>
<td>Secure Training</td>
</tr>
</tbody>
</table>
Now, if we click on the icon “Assign Resources” from the main menu, MS Project brings up a small table listing all the resources assigned to each task and the current effort being expended. If we change the “units” from 100% to 200% and click on “Assign,” the expected time for the activity is cut in half (3 wks → 1.5 wks) as shown in Figure 14.34.

One of the nicest features available in MS Project is the ability to keep track of the costs on the project. In order for it to do this, we first must enter all available information (like the data presented in Table 14.5 of Section 14.7). To enter this information, click on View, then Resource Sheet. This brings up a blank table for you to enter each of the resources and what their costs are (see “Resource Name” and “Std. Rate” columns of Figure 14.35). Again, in this example, we identified earlier eleven resources that have a direct 1:1 correspondence with the eleven activities (e.g., “Construct” is the resource that goes with the “Construct Interior” task). We have simply entered their costs (on a weekly basis) according to the information in Table 14.5 (e.g., task 1 has a cost of $2,100 for a 3-week duration, so we enter it as $700/wk) and indicated that we want the costs accrued at a “prorated” basis (which is the MS Project default choice). Other possible choices include full billing of the cost upon completion of the task, or at the start of the task. Once this data is completely entered, it should look as shown in Figure 14.35.

Of even greater value is the ability to go into the charts on a periodic basis and update what percentage of the different activities are completed and see what costs should have accrued according to the prorated budget. To demonstrate this ability, we’ll use column (1)
of Table 14.8. We’ll just demonstrate how to enter the % complete for one activity and let
the student enter the remaining ten activity completion %’s. First, you must highlight the
activity for which you want to enter some information. For our example, let’s choose activ-
ity A (Select Office Site). Then click on Tools, Tracking>, and then Update Tasks and MS
Project will bring up a menu as shown in Figure 14.36. We simply enter the information
requested in the “% Complete” box. Once we’ve done this for all eleven activities, we can
go back to see the cost information that MS Project has now calculated for us (by clicking
on Project, Project Information, then Statistics) as shown in Figure 14.37. So far it says we
should have spent $18,500 of the budgeted total of $90,500, with about 45% of the actual
project duration (days) being completed. Obviously, we have the more expensive tasks still
ahead of us. However, it may not be obvious that in order to calculate the actual and
remaining costs, MS Project has assumed that each activity is coming in right on its budget
(i.e., no cost variance at all). Real project managers would recognize that this is “too rosy”
of a scenario, so let’s look at how we can tell the software what the actual costs have come in
at. To update the actual cost figures, we must do the following:

1. On the Tools menu, click Options, and then click on the Calculation tab.
2. Clear the “Actual costs are always calculated by Microsoft Project” check box.
3. Click OK.
4. Click View, then Task Usage.
5. Click View again, point to Table: (default name)>, and then click Tracking.
6. On the Format menu, point to Details>, and then click Actual Cost.
This will give you a table as shown in Figure 14.38 and you simply enter the actual costs to date (using the values in column (4) of Table 14.8) for each activity as shown. For example, for task 1 (Select Office Site), the budget was originally $2,100 (see Figure 14.35), but it actually came in at $2,300 (a $200 cost overrun). MS Project is now able to calculate the updated cost information, which is shown in Figure 14.39 (to see it, click on Project, Project Information, then Statistics). Here we see that the actual total costs to date are $22,100, and that the total project budget (Current Cost) is now projected to be $95,600, rather than the baseline estimated cost of $92,000—a cost overrun of $3,600. As you can see, this can be a very powerful tool in helping a project manager stay on top of both the progress and the costs to date. The completed MS Project file is saved on the student disk as GLOBALREV.MPP.
Some 40 years after its inception, the critical path concept is an important part of current practice. Almost any time you have a large project with a number of interrelated activities, you will find a network-based planning and reporting system being used. Over time, the distinctions between PERT and CPM have become blurred. Firms have developed their own internal computer-based models incorporating those features of the original PERT and CPM systems that are important to their specific activity. EDS (Electronic Data Systems) has been using AutoCAD (an automatic drafting program) to draw PERT charts for clients for several years now. Managers find that this visual aid helps them and their clients to keep projects on track, to spot possible problems early on, and acts as a good communication device. The charts are printed across several pages, making a large “banner” that can be posted in the office.

It is common to think of using critical path methods on large one-of-a-kind projects (e.g., the U.S. Space Shuttle). However, the methods can and do apply to activities that occur at quite regular intervals. A good example of such an activity is the major overhaul of the dragline in a particular coal mining operation. This maintenance must be performed on a fairly regular schedule and is of major importance because when the dragline is not operating, the mine is not operating. The firm can and does use the network diagram for the previous overhaul in planning for an upcoming overhaul. The network plays an important role in assuring that all those involved understand the various steps and their interrelationship, as well as ensuring that all the requisite parts and materials are available when they are needed. Given the turnover in personnel, and human frailties, the network serves as a convenient way to capture past experience. The activities associated with the overhaul vary from time to time, so the chart must be reworked, but the previous version generally provides a good starting point.

For our example firm, estimating time variability is not an important part of creating its PERT-CPM network. Indeed, it relies on a single best estimate rather than the three that are part of the PERT approach. The crucial element in developing an economical plan for the overhaul is the availability of various skilled workers (electricians, pipefitters, and so on). Each time an overhaul is to be performed, the tasks vary somewhat, and thus the demand for workers varies from year to year. In addition, the supply of available workers within the firm varies from time to time depending on the level of other activities. The planning operation typically involves running the model under a variety of assumptions. Alternatives might include regular employees working regular time, regular employees doubling up (i.e., working two shifts), bringing in outside workers, and so on. Such calculations can make it clear, for example, that it pays to spend $20,000 in overtime for electricians if it puts the mine back in operation a day earlier.

The computer has had a major impact on the use of CPM and PERT. Large construction projects may require a thousand or more nodes. In the 1960s it was not unusual to find the network diagram for such a project spread out over three walls of a room that was dedicated to that purpose for the duration of the project. Major changes in the plan were a major pain in their own right, and communication among the multiple contractors was cumbersome. The computer has changed all of that. The analysis is now done on a computer. There is a whole host of microcomputer software (e.g., Harvard Project Manager, InstaPlan, MacProject II, Microsoft’s Project for Windows [demonstrated in Section 14.8]) that will aid in project management, combining PERT and CPM with budget information and control. It is quite a simple matter to use the software to draw the boxes (actually combining activities and events) and then to connect them by pointing to the activities that precede each other. The program does the rest (determining the critical path, earliest and latest starting and finishing times). Multiple runs spanning the life of the project are the order of the day. In the early phases it is important to ensure that orders for
major components are placed early enough. Complicated systems (i.e., generators, furnaces, and so on) may have a delivery time of several years. Regular updating based on supplier reports enables management to see when it is necessary to expedite an order. New information is fed into the model, and the program is rerun on a weekly basis. Obviously, the information obtained from such runs influences the allocation of resources. It may even affect the design of the project. If a cost control report more or less like the one shown in Table 14.8 indicates serious cost overruns early enough, later parts of the project may be redesigned. For example, one firm reported reworking a heating plant design based on one boiler rather than two after the cost of excavation and driving piles ran far ahead of budget. This change enabled the firm to bring the project in on time and within budget.

This chapter dealt with the role of PERT and CPM in project management. The fundamental concept is to represent a project as a network. Section 14.2 showed how to use an activity list to construct a network diagram for a project, where the activity list identifies each activity in the project and its immediate predecessors. Section 14.3 showed how the network diagram and the expected activity times are used to determine the critical path, which is a longest path through the network. In the process, the terms earliest start time, earliest finish time, latest start time, latest finish time, and slack were defined.

Section 14.4 introduced the notion of variability in activity times. It dealt with two main topics: the PERT system of estimating times and the probability that all the activities on the critical path will be completed by a specified date. The PERT system of estimating time is based on the assumption that activity time has a beta distribution. It uses an optimistic, a most probable, and a pessimistic time estimate to derive the expected activity time and the standard deviation of the activity time.

Management would like to know the probability that the project under consideration will be completed by a specific date. If one assumes that activity times are independent and that the sum of the activity times on the critical path has a normal distribution, it is a straightforward exercise to calculate the probability that the critical path will be completed by a specified date. This is not the probability that the project will ultimately be completed by the specified date, for the effect of randomness could turn a supposedly noncritical path into a critical one. However, this does give an upper estimate for the probability that the overall project will be completed by a specific date.

Section 14.6 presented the CPM framework for analyzing the problem of time-cost trade-offs. The amount of time that an activity takes is determined by the level of resources devoted to that activity. The model in this section employs the notion of project crashing. The model is intended to help management select a completion time for each activity so as to achieve a specified completion date for the overall project at minimum cost. The basic input for the model is a set of functions, one for each activity. Each function portrays the activity cost as a linear function of activity time within specified limits on the time. These data are then used either in a marginal cost analysis or in a linear programming model to select the best activity times.

Section 14.7 considered project cost management via the PERT/Cost system. It dealt with both a cost planning and a cost control model. The planning model produces a graph of the feasible budget demands as a function of time. This graph is constructed from the resource usage profiles based on the earliest start time and the latest start time.

The project cost control model is a system of comparing actual costs with budgeted costs. The budgeted cost model uses the assumption that for partially completed activities, the budgeted cost is equal to the budget for the completed activity multiplied by the proportion of the activity that has been completed. The model allows management to recognize cost overruns on various activities before they are completed.
Section 14.8 demonstrated the use of the valuable software package, MS Project for Windows, to carry out most of the techniques covered in the previous sections. This software greatly simplifies the process of generating links between tasks and giving us a graphical view of the critical activities and the entire project. Certainly, it is easier to manage a complex project with the use of such software compared to doing it by hand or in a spreadsheet.

**Key Terms**

**PERT.** An acronym for Program Evaluation Review Technique, a method for scheduling and controlling projects.

**CPM.** An acronym for Critical Path Method, a method for scheduling and controlling projects.

**Activity List.** A list of jobs in a project with their immediate predecessors, expected times, and resources required.

**Immediate Predecessors.** Those activities that must be completed immediately prior to the start of the activity in question.

**Network Diagram.** Graphical method of representing a project with nodes and arcs.

**Branch.** A line in a PERT network indicating an activity (in AOA) or precedence (in AON). Also called an arc.

**Node.** A circle in a PERT network indicating the completion of certain activities and the initiation of others (in AOA) or the activity itself (in AON).

**Event.** The completion of all activities leading into a node in a PERT network using AOA.

**Activity.** A job that must be completed as part of a project.

**Dummy Activity.** An imaginary activity that requires no time and is used to maintain the appropriate precedence relationships in a PERT network diagram using AOA.

**Path.** A sequence of activities leading from the starting node to the completion node of a network.

**Critical Path.** A sequence of activities that determines the longest path through the network that yields the minimum time in which an entire project can be completed.

**Critical Activities.** The activities on the critical path.

**Earliest Start Time.** In a PERT network, the earliest moment at which an activity can start.

**Earliest Finish Time.** In a PERT network, the earliest moment at which an activity can be completed.

**Forward Pass.** The process of moving along a network from beginning to end, computing the earliest start time and earliest finish time for each activity.

**Backward Pass.** The process of moving backward along a network from end to beginning, computing the latest finish time and then the latest start time for each activity.

**Latest Start Time.** In a PERT network, the latest moment at which an activity can start without delaying completion of the overall project.

**Latest Finish Time.** The latest time at which an activity can be completed without delaying the completion of the overall project.

**Slack.** The time that an activity can be delayed beyond its earliest start time without delaying the completion of the overall project.

**Optimistic Time.** The time required to complete an activity if everything goes perfectly.

**Most Probable Time.** The time required to complete an activity under normal circumstances.

**Pessimistic Time.** The time required to complete an activity under the most unfavorable conditions.

**Beta Distribution.** A probability distribution used to model the activity times in PERT.

**Normal Time.** In CPM, the maximum time for completion of an activity, corresponding to minimal resource usage.

**Normal Cost.** The cost required to achieve the normal time.

**Crash Time.** In CPM, the minimum possible time for completion of an activity, corresponding to maximal resource concentration.

**Crash Cost.** The cost required to achieve the crash time.

**Crashing.** A term in the CPM method describing the process of reducing the time required to complete an activity.

**PERT/Cost.** A system for determining the feasible patterns of cash flow during a project.

**Self-Review Exercises**

**True-False**

1. **T F** In a PERT network diagram using AOA, each activity is represented by a circle called a node.

2. **T F** The term event is used to refer to nodes in a PERT network using AOA.

3. **T F** A dummy activity is required in a correct network representation of the following activity list.

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>IMMEDIATE PREDECESSORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2, 3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>
4. \( T \ F \) The earliest finish time for an activity depends on the earliest finish time for the project.

5. \( T \ F \) The latest finish time for an activity depends on the earliest finish time for the project.

6. \( T \ F \) All activities on the critical path have their latest finish time equal to their earliest start time.

7. \( T \ F \) A strategic analysis of a PERT network concentrates on the allocation of resources to reduce the time on the critical path.

8. \( T \ F \) The probability of completing the project by time \( T \) is equal to the probability of completing the critical path by time \( T \).

9. \( T \ F \) The standard deviation of an activity time is estimated as \( (b - a)/6 \), where \( b \) is the pessimistic and \( a \) is the optimistic time.

10. \( T \ F \) The CPM approach to time-cost trade-offs assumes that cost is a linear function of time.

11. \( T \ F \) The LP formulation of the network-crashing problem minimizes the total cost of crashing subject to an upper bound on project duration.

12. \( T \ F \) In the PERT/Cost model the earliest starting time total cost schedule always is less than or equal to the latest starting time total cost schedule.

13. \( T \ F \) Time variabilities leading to a longer-than-expected total time for the critical path will always extend the project completion date.

14. \( T \ F \) If a noncritical activity is delayed more than its slack time, all other factors remaining unchanged, then the project completion date will be extended.

15. \( T \ F \) Gantt charts provide useful immediate predecessor information.

Multiple Choice

16. Of all paths through the network, the critical path
   a. has the maximum expected time.
   b. has the minimum expected time.
   c. has the maximum actual time
   d. has the minimum actual time

17. The earliest start time (ES) for an activity leaving node C (in the AOA approach)
   a. is the Max of the earliest finish times for all activities entering node C
   b. equals the earliest finish times for the same activity minus its expected activity time
   c. depends on all paths leading from the start through node C
   d. all of the above

18. The latest finish time (LF) for an activity entering node H (in the AOA approach)
   a. equals the Max of the latest start times for all activities leaving node H
   b. depends on the latest finish time for the project
   c. equals the latest start time minus the activity time for the same activity
   d. none of the above

19. The slack for activity G
   a. equals LF for G – LS for G
   b. equals EF for G – ES for G
   c. equals LS for G – ES for G
   d. none of the above

20. Estimating expected activity times in a PERT network
   a. makes use of three estimates
   b. puts the greatest weight on the most likely time estimate
   c. is motivated by the beta distribution
   d. all of the above

21. The calculation of the probability that the critical path will be completed by time \( T \)
   a. assumes that activity times are statistically independent
   b. assumes that total time of the critical path has approximately a beta distribution
   c. requires knowledge of the standard deviation for all activities in the network
   d. all of the above

22. In the CPM time-cost trade-off function
   a. the cost at normal time is 0
   b. within the range of feasible times, the activity cost increases linearly as time increases
   c. cost decreases linearly as time increases
   d. none of the above

23. The marginal cost of crashing a network could change when
   a. the activity being crashed reaches its crash time
   b. the activity being crashed reaches a point where another path is also critical
   c. both a and b

24. Fundamental ideas in the LP network crashing models are
   a. activity time equals normal time + crash time
   b. earliest start time for an activity leaving a node equals the Max of the earliest finish times for activities leaving that node
   c. earliest finish time equals latest finish time minus activity time
   d. none of the above
25. The PERT/Cost model assumes that
   a. each activity achieves its optimistic time
   b. the costs are uniformly distributed over the life of the activity
   c. activity times are statistically independent
   d. none of the above

26. The PERT/Cost control report
   a. requires a budget for each activity
   b. requires a report on the percentage of completion of each activity
   c. calculates cost overruns
   d. all of the above

Answers

Skill Problems
14-1. The Build-Rite Construction Company has identified ten activities that take place in building a house. They are
   1. Walls and Ceiling (erect the wall frames and ceiling joists).
   2. Foundation (pour the foundation slab).
   3. Roof Timbers (put up the roof timbers).
   4. Roof Sheathing (install roof sheathing over the timbers).
   5. Electrical Wiring (install the electrical wiring).
   6. Roof Shingles (shingle the roof).
   7. Exterior Siding (put on the exterior siding).
   8. Windows (install the window units).
   9. Paint (paint the interior and exterior).
   10. Inside Wall Board (hang the inside wall board).
In addition, the following customs are typically observed:
   1. Wiring is done from the interior side of the wall, while the window unit is mounted after the wall frame has been erected.
   2. Inside wall board and exterior siding are installed over the window unit.
   3. Painting is not begun until the house is watertight.
Make a list showing each activity and its immediate predecessors (use MS Project if available).

14-2. Quacker Mills hires engineering undergraduates and moves them through six management experiences (activities) to prepare them to be plant managers. There are three disciplines and two positions (starting and advanced) in each discipline. These six activities are shown in Table 14.9. Further, a person who has not been a production line engineer cannot be department head, nor can someone who has not been a foreperson be a product line scheduler. Make an activity list showing each activity and its immediate predecessors (use MS Project if available).

14-3. Construct the network diagram using activity-on-the-arc for the house construction system used by Build-Rite Construction Company in Problem 14-1.

14-4. Construct a CPM network diagram (AON) of the 12 activities given in Table 14.10 (use MS Project if available).

Table 14.9

<table>
<thead>
<tr>
<th>DISCIPLINE</th>
<th>Plant Engineering</th>
<th>Line Supervision</th>
<th>Production Planning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting</td>
<td>Engineering 1</td>
<td>Foreperson 2</td>
<td>Assistant Product 3</td>
</tr>
<tr>
<td>Advanced</td>
<td>Plant Engineer 4</td>
<td>Department Head 5</td>
<td>Line Scheduler</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Product Line Scheduler 6</td>
</tr>
</tbody>
</table>
14-5. Build-Rite has estimated the times given in Table 14.11 as necessary to complete each of the tasks involved in building a house. For each activity, give the
(a) Earliest start time
(b) Earliest finish time
(c) Latest start time
(d) Latest finish time
(e) Slack
In addition, identify the critical path (use MS Project if available).

14-6. As a project manager, you are faced with the activity network and estimated activity times shown in Figure 14.40. For each activity, give the
(a) Earliest start time
(b) Earliest finish time
(c) Latest start time
(d) Latest finish time
(e) Slack
In addition, identify the critical path (use MS Project if available).

**FIGURE 14.40**
14-7. You are called in as a production consultant. The plant currently uses a PERT-CPM approach to a production run described by the activity network of Figure 14.41. Based on your evaluation, however, the immediate predecessors of each activity are as follows:

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>IMMEDIATE PREDECESSORS</th>
<th>ACTIVITY</th>
<th>IMMEDIATE PREDECESSORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>—</td>
<td>H</td>
<td>E</td>
</tr>
<tr>
<td>B</td>
<td>—</td>
<td>I</td>
<td>G</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>J</td>
<td>E</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>K</td>
<td>H</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
<td>L</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>M</td>
<td>I, J, K, J</td>
</tr>
<tr>
<td>G</td>
<td>D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Draw the revised activity network.
(b) Compute earliest and latest start and finish times for the revised network based on the assumption that each activity takes 1 hour longer than its alphabetic predecessor (i.e., A = 1 hour, B = 2 hours, etc.).
(c) Find the slack of each activity and identify the critical path (use MS Project if available).
(d) How much less time does the production run take under this revised activity network than it did with the original network?

14-8. On the basis of company history, Build-Rite’s management has determined that the optimistic, most probable, and pessimistic times for each activity are as follows:

<table>
<thead>
<tr>
<th>ACTIVITY NUMBER</th>
<th>ACTIVITY</th>
<th>OPTIMISTIC TIME (DAYS)</th>
<th>MOST PROBABLE TIME (DAYS)</th>
<th>PESSIMISTIC TIME (DAYS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Walls and Ceiling</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Foundation</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Roof Timbers</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Roof Sheathing</td>
<td>1</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Electrical Wiring</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Roof Shingles</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>Exterior Siding</td>
<td>1</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Windows</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Paint</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>Inside Wall Board</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Compute the expected activity time and the standard deviation for each activity (use MS Project if available).
(b) Print out a copy of the pessimistic Gantt chart if MS Project is available.

14-9. On the basis of the activity network shown in Figure 14.42 and the associated activity times given below, compute the expected value and the standard deviation for each activity time.
(a) Find the earliest start times, earliest finish times, latest start times, latest finish times, and slack for each activity.
(b) Specify the critical path (use MS Project if available).
14-10. Assume that activity times in Build-Rite’s activity network (see Problems 14-1, 14-5, and 14-8) are independent of each other and that the sum of any combination of activity times is normally distributed.

(a) What is the probability that all the activities on the current critical path will be completed within 12 days?

(b) Within 25 days?

(c) Is this the same as the probability that a house will be completed in 25 days? Comment.

14-11. For the network in Problem 14-9, answer the following questions:

(a) Under the usual assumptions, find the probability that the activities on the critical path will be completed within 20 weeks.

(b) How many weeks should be allowed to give a 95% probability of completing the critical path on time?

14-12. Build-Rite’s engineers have calculated the cost of completing each activity in both normal time and crash time, where the values for normal time and crash time, respectively, correspond to the estimates of expected time and optimistic time from Problem 14-8. Their results are given below.

(a) Specify the normal time, normal cost, crash time, crash cost, maximum crash days, and cost per crash day for each activity. Assume linear cost relationships.

(b) Compute the expected cost of the project (based on normal time).

(c) Suppose that the company has to reduce the completion time by seven days. How much would this reduction cost?

(d) How much would it cost to reduce the completion time by eleven days?
14-13. Use the normal costs from Problem 14-12 and the time data from Problem 14-5 to construct early-start and late-start cost tables and the graph of cumulative expenditures versus time for Build-Rite.

14-14. The resources required for the activities in Problem 14-6 are given below. Construct early-start and late-start cost tables, and graph cumulative budget demands versus time, for both schedules (use MS Project if available).

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>TOTAL RESOURCES REQUIRED ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,800</td>
</tr>
<tr>
<td>B</td>
<td>3,000</td>
</tr>
<tr>
<td>C</td>
<td>900</td>
</tr>
<tr>
<td>D</td>
<td>3,000</td>
</tr>
<tr>
<td>E</td>
<td>6,000</td>
</tr>
<tr>
<td>F</td>
<td>12,000</td>
</tr>
<tr>
<td>G</td>
<td>3,200</td>
</tr>
<tr>
<td>H</td>
<td>3,200</td>
</tr>
<tr>
<td>I</td>
<td>7,200</td>
</tr>
<tr>
<td>J</td>
<td>7,000</td>
</tr>
<tr>
<td>K</td>
<td>2,400</td>
</tr>
<tr>
<td>L</td>
<td>2,000</td>
</tr>
<tr>
<td>M</td>
<td>4,500</td>
</tr>
</tbody>
</table>

14-15. Build-Rite’s record of historical expenditures at the end of day 15 is as follows:

<table>
<thead>
<tr>
<th>ACTIVITY NUMBER</th>
<th>ACTIVITY</th>
<th>COST INCURRED TO DATE ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Foundation</td>
<td>22</td>
</tr>
<tr>
<td>1</td>
<td>Walls and Ceiling</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
<td>Roof Timber</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Roof Sheathing</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Roof Shingles</td>
<td>4.50</td>
</tr>
<tr>
<td>5</td>
<td>Electrical Wiring</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>Windows</td>
<td>22.50</td>
</tr>
<tr>
<td>10</td>
<td>Inside Wall Board</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Exterior Siding</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>Paint</td>
<td>0</td>
</tr>
</tbody>
</table>

14-16. Review the data for Problem 14-14. Prepare an analysis of the cost of the project to date if the figures below represent the status as of the end of the eighth unit of time (use MS Project if available). Assume that the budgeted cost is equal to the budget for the completed activity multiplied by the proportion of the activity that is complete.
Application Problems

14-17. Consider the network and activity times shown in Figure 14.43. You would like to reduce the minimum time to complete the project. Suppose that you can reduce an activity time as much as you like as long as you increase some other activity or activities by the same amount. For example, you can reduce G by 1 hour if you increase C and D by \( \frac{1}{2} \) hour each. Assume that activity times of zero are permissible.

(a) Find the current critical path and the minimum time required to complete the project (use MS Project if available).

(b) Reallocate times to achieve the minimum possible time to complete the project (use MS Project if available). Note that in this network, the total of all activity times must equal the current total of 20 hours.

14-18. Consider the activity network in Problem 14-9. The following are estimates of costs for completion in crash time and normal time, where the times correspond to the optimistic and expected time, respectively:

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>CRASH COST ($)</th>
<th>NORMAL COST ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>E</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>F</td>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>G</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>H</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>I</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>J</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Prepare a table showing the normal time, normal cost, crash time, crash cost, maximum crash weeks, and cost per crash week for each activity.

(b) What would be the minimum cost of the project if it were to be completed in
   (i) 19 weeks?
   (ii) 18 weeks?
   (iii) 17 weeks?

(c) Formulate a linear programming model that will assess the additional cost of reducing the completion time to 9 weeks. What is the best way to accomplish the reduction? What's the additional cost?

14-19. Refer to Problems 14-3 and 14-12 and formulate a linear programming model that would allow Build-Rite to assess the cost of crashing its activity network by \( x \) hours.
14-20. Consider the activity network and normal activity times shown in Figure 14.44, as well as the data in the following table.

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>COST/CRASH UNIT ($)</th>
<th>MAX CRASH UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>E</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>—</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Find the critical path and the minimum time required to complete the project.
(b) Prepare a table showing which activities to crash as project length decreases.

14-21. This is a more complex version of Problem 14-17. As the production supervisor for the Hurricane Fan Company, you have used PERT-CPM techniques to schedule your production runs. Your current activity network and activity times are shown in Figure 14.45. A production consultant has pointed out that, due to the similarity in job skills needed for each activity, resources are perfectly transferable between activities (i.e., the time required to do an activity can be reduced by any amount by increasing the time required for another job by the same amount). If the consultant is correct, how much can the time needed for each production run be reduced?

14-22. This problem is a further examination of the PERT assumptions. Consider a project with five activities and the precedence relationships given in the table below. The Range column indicates the possible range of days it might take to complete an activity.

(a) Suppose that any time within the range is equally likely (i.e., a uniform distribution). Use the standard PERT analysis and compute the expected project length. Also calculate the probability that the project will be completed in the expected project length + 5 days. Use the fact that if the range is $a$ to $b$ then the mean time is \( \frac{a + b}{2} \) and the variance of the time is \( \frac{(b - a)^2}{12} \).
(b) Now set up a spreadsheet to simulate the project (using Crystal Ball or @Risk if available). Simulate 100 trials and compute the mean project length. Is it larger or smaller than your previous answer? Why?

(c) What is the probability that the project will be completed before the expected time from part (a) + 5 days. Is it larger or smaller than your previous answer? Why?

14-23. This problem is the same as Problem 14-22, but now assume that the activity times are normally distributed with the following means and standard deviations:

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>11.5</td>
</tr>
<tr>
<td>B</td>
<td>27</td>
<td>1.15</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>11.5</td>
</tr>
<tr>
<td>D</td>
<td>27</td>
<td>1.15</td>
</tr>
<tr>
<td>E</td>
<td>27</td>
<td>1.15</td>
</tr>
</tbody>
</table>

(a) Use the standard PERT analysis and compute the expected project length. Also calculate the probability that the project will be completed in the expected project length + 5 days. Is your answer the same as in part (a) of Problem 14-22? Why?

(b) Now set up a spreadsheet to simulate the project (using Crystal Ball or @Risk, if available). Simulate 100 trials and compute the mean project length. Is it larger or smaller than your previous answer? Why?

(c) What is the probability that the project will be completed before the expected time from part (a) + 5 days. Is it larger or smaller than your previous answer? Why?

(d) How sensitive is the simulation analysis to the form of the activity time distribution (uniform in Problem 14-22, normal in Problem 14-23)?

References
